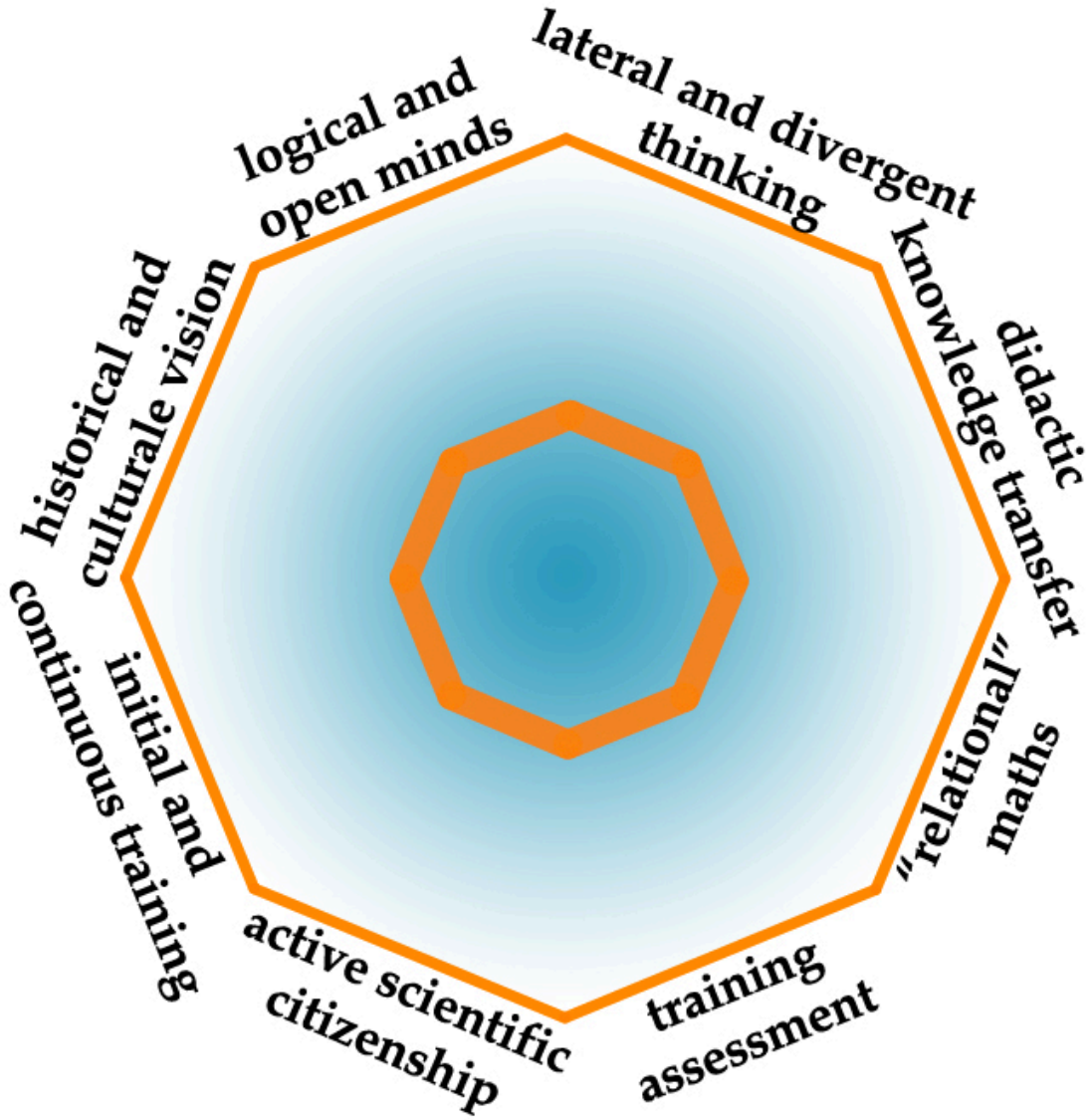
MANIFESTO ABOUT MATH TEACHING

For a training, democratic, responsible use of math thinking and its tools according to Italian "Indicazioni nazionali e nuovi scenari" (2018).

JUNE 2020



Why has a Manifesto about math teaching been conceived?

In order to allow students at every school level to:

- use math tools to understand reality and to be aware of real perceived data;
- build up their cognitive structures to achieve active citizenship
- promote an experimental attitude in order to overcome the distrust towards maths.

In order to allow teachers :

- to develop interest in the research for math teaching and learning;
- to experiment group exchanges of teaching methods;
- not to accept easy but unsatisfactory solutions;
- to discover and value their learning and planning skills.

This manifesto is meant for teachers, headteachers, teachers' trainers, parents, researchers and students.
The starting point of the discussion relating to the mathematic groups of the MCE is the ministerial document "Indicazioni nazionali e nuovi scenari",
where "math thinking" is considered a cultural element for citizenship.

We can highlight the following items:

- in math learning we can see differences in learning according to gender, rhythm, times and cultures;
- maths is an obstacle for many students and can be a means of school and social selection;

Thanks to the **"Four steps for an emancipation Pedagogy"** campaign the MCE promotes active practices according to this Manifesto.

Aims and didactic strategies

1 Promoting a **historical and cultural vision of maths**, as a means of knowledge and organization of reality, bearing in mind students' native culture.

a) Creation of **motivating contexts** (1) avoiding unreal situations which have a meaning only to teachers
b) Working on words and symbols **to move from natural language to mathematic language** with a proper gradual use of the meanings
c) **Enhancing logic, linguistic, cognitive structures** to offer students the right tools in different social environments
d) Comparison among **different ways of quantifying, comparing, classifying, measuring and calculating** in accordance with different cultures (ethnomathematics) (3).

2 Training **logical and open minds** which are able to use math structures in order to quantify, discuss, criticize, justify, face problems, and ask questions with a rational mind even in uncertain situations.

a) Creating a **learning environment** to support students when they face understanding difficulties and frustration.
b) Using **math discussion** as a way to negotiate meanings and shared knowledge.
c) Considering **mistakes a productive step** in the learning process and a resource for the individual and groups: helping students correct themselves without accepting right answers.

3 Enhancing, educating and developing flexibility and **lateral and divergent thinking** (4) in order to

avoid difficulties and negative emotions, increasing pleasure, curiosity, fun and developing creativity.

a) Creation of **a math workshop** where students can experiment, discover and build up knowledge thanks to an active teaching method.
b) Valuing **the role of the group and the class** as a place where students "inventions" are shared and interpreted in different ways.
c) In collaboration with students **material, symbolic, graphic, digital "models"** to be manipulated can be built to endeavour understanding and conceptualization processes.
d) **Using digital devices** to support generalization processes and thinking about relationships among math items also as regards weak subjects.
e) Research for **math rules in nature** and observing natural things to discover math concepts.

4

Being aware of **knowledge transfer**, elaborating and experimenting effective didactic strategies thanks to careful planning and keeping in mind students' cognitive processes

a) Building up math concepts with a **'helicoidal' didactic**, reinforcing and developing themes and concepts slowly in the long run.
b) Giving a meaning to traditional math items through **guided reinventions** of underlying concepts.
c) Being aware of relationships between the body and "math items" (**embodiment**) (8) is needed to reach your mind through your physical experiences.
d) Taking care of **learning processes** to avoid the creation of **obstacles and misconceptions.**
e) Taking care of the **organization of "school spaces"** to enhance students' habits and interests.

5

Focusing on a **"relational" maths** based on discovering the relationship among "mathematics objects" ('why, 'how', explanations,…)

a) Research and recognition of rhythm, structures, cycles, regularities in real situations and their **formal math structures** through more and more complex tools (algebra, functions, geometric representations)
b) **Modelling** of real situations to understand meanings and practical utility of maths
c) Developing the ability to see through **'the eyes of the mind'** what is not easy to perceive in real life in order to perceive abstract concepts
d) Creation of **mind maps** to organize abstract concepts in a network of meanings (11).

6

Achieving a **training assessment** based on a survey of the pupils' learning that takes into account the implicit competences already present in the children and their evolution to obtain useful elements for the continuous revision of the didactic paths, calibrating them on the actual learning outcomes.

a) Use of **qualitative** tools such as: observations records, journals, documentations, photos, videos, record of discussions,..
b) **Self-assessment** thanks to students' feedback

7

The use of maths in order to build up an **active scientific citizenship** through a democratic participation (listening, conveying and justifying one's ideas).

a) Knowledge and use of **different codes** (graphic, linguistic, iconic, symbolic codes) in order to express math thinking).
b) Critical use of **school texts and alternative materials** (websites, videos, structured materials, forms, exercices).
c) Reading and **critical interpretation of information** coming from the media (graphs, percentages, fractions,..)

8 Getting deeper into math concepts for **initial and continuous training** according to the different school levels.

a) Participation in **research groups** to discuss and share contents and methodology according to the results of math research.
b) Organization of **didactic experiments** with tutors to prepare **activities** for classes
c) Backward planning to draw back to the **origin of maths concepts**.
d) Organization of **meetings with schools and families** to inform them about the results of the researches in order to overcome stereotypes concerning maths.

Proposals

This concept of maths implies some choices regarding teaching contents and methods so that students can achieve competencies not only for their school curricula but also for themselves and their lives and for society (Emma Castelnuovo).

In compliance with this Manifesto teachers have to:

- apply the Manifesto principles to real and coherent actions sharing personal experiences with proper documentation
- collaborate to the spreading of group results
- participate in **seminars** on specific topics
- carry out proper actions to fight against sociocultural gap among the different areas of each country, being aware of education poverty and discrimination.

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Notes and insights

(1) **Avoid exercises which refer to stereotypes** and have no meaning to students. Start from visible objects and experiences.

(2) Starting from the natural language it is important to create a "**familiar language**" belonging to the group, some definitions formed in the class. Later on, when a concept is acquired, specific traditional math language can be used.

(3) According to D'Ambrosio **ethnomathematics** means recognizing that every culture develops different ways of explaining, knowing, and facing reality. In every culture this research leads to quantifying, comparing, classifying, measuring which generates maths. (Mendes, 2008, p. 19: "etnomathematics can be considered a knowledge area connected with cultural groups, their interests, conveyed through an ethnic language influenced by the group culture.") Rosa and Orey (2006) state that "etnomathematics is the way specific cultures have developed techniques and ideas through history in order to deal with measures, calculations, comparisons, classifications and different ways of explaining natural events.

(4) Joy Paul Guilford, American psychologist, has outlined two opposite, different styles of thinking. **Convergent thinking** make people find a unique solution when facing a problem. Math problems are based on this logical thinking: there is only one solution. On the contrary, **divergent thinking** is characterized by the research for multiple solutions. Edward De Bono, Maltese psychologist, shows a way of problem solving called **'lateral thinking'**: looking at the problem from different perspectives.A direct solution uses sequential logical thinking referring to obvious considerations, whereas lateral thinking searches for alternative points of view.

(5) It is necessary to learn how to use scissors, to light a match, to tie knots, to pour liquid substances. Dexterity must be trained like perception, at every age. Activities carried out in a workshop are engaging.

(6) **Helicoidal didactic** is the ability of renforcing themes and concepts, widening them.

(7) We propose a **slow structured didactic in the long run** even if today's society considers speed an absolute value. Slowness allows people to focus on mistakes, difficulties, obstacles.

(8) Since the eighties G. Lakoff, M. Johnson and others authors have developed some studies on embodied aspects of cognitive and mental processes. The research has developed in different fields (from cognitive linguistic to artificial intelligence, from neurobiology to phenomenology as well as in the studies on metaphors.). *Embodiment* means incorporating concepts and experience them in the body to understand and give meaning to maths.

(9) Many examples refer to space metaphors expressed by every day gestures and words. For example keeping your arms forward and looking ahead to show the future. The rigid structure of the classroom prevent teaching maths from using the body and allows a taught class. On the 2030 Agenda -the European Commission has inserted the fight against sedentary lifestyle among its aims.

(10) **Math modelling** is a cognitive process leading to building up a simplified model of reality that can be transferred through math schemes e.g. a calculation, an expression, an algebra formula, a graph,…

(11) Gerard Vergnaud's **theory of conceptual fields:** a group of problems and situations dealt with different types of concepts, procedures and representations which are strictly connected, e.g. the multiplying structures…

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