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How counting represents number: What children must learn and when they learn it ☆

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ABSTRACT

This study compared 2- to 4-year-olds who understand how counting works (*cardinal-principle-knowers*) to those who do not (*subset-knowers*), in order to better characterize the knowledge itself. New results are that (1) Many children answer the question "how many" with the last word used in counting, despite not understanding how counting works; (2) Only children who have mastered the cardinal principle, or are just short of doing so, understand that adding objects to a set means moving forward in the numeral list whereas subtracting objects mean going backward; and finally (3) Only cardinal-principle-knowers understand that adding exactly 1 object to a set means moving forward exactly 1 word in the list, whereas subset-knowers do not understand the unit of change.

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1. Introduction

It seems uncontroversial to say that some children know how to count and others do not. But how can we tell them apart? If knowing how to count just means reciting the numeral[1] list (i.e., "one, two, three...") up to "five" or "ten," perhaps pointing to one object with each numeral, then many two-year-olds count very well (Baroody & Price, 1983; Briars & Siegler, 1984; Fuson, 1988; Fuson, Richards, & Briars, 1982; Gelman & Gallistel, 1978; Miller & Stigler, 1987; Schaeffer, Eggleston, & Scott, 1974). That kind of counting is good for marking time (e.g., *close your eyes and count to ten...*) or for playing with one's parents, but reciting the alphabet or playing patty-cake would do just as well. The thing that makes counting different from the alphabet or patty-cake is that counting tells you the number of things in a set.

Of course, counting only tells you this if you do it correctly, following the three 'how-to-count' principles identified by Gelman and Gallistel (1978). These are (1) The one-to-one principle, which says that "in enumerating a set, one and only one [numeral] must be assigned to each item in the set." (p. 90); (2) The stable-order principle, which says that "[Numerals] used in counting must be used in the same order in any one count as in any other count." (p. 94); and (3) The cardinal principle, which says that "the [numeral] applied to the final item in the set represents the number of items in the set." (p. 80).

As Gelman and Gallistel pointed out, so long as the child's counting obeys these three principles, the numeral list ("one," "two," "three,"... etc.) represents the cardinalities 1, 2, 3,... etc. The relation of numerals to cardinalities is governed by the successor function: If numeral "*N*" represents cardinality *N*, then the next numeral on the list represents the cardinality $N+1$, which is the successor of *N*. The counting principles are what make counting equivalent to saying "one, (plus one is) two, (plus one is) three,..."

In their 1978 book, Gelman and Gallistel argued that even 2-year-olds honor these principles when counting, because the principles are intuitively understood. This view has come to be called the *principles-first* (or *principles-before-skills*) view.

Other studies, however, have failed to provide support for the principles-first view. For example, three-year-old children often violate the one-to-one principle by skipping or double-counting items, or by using the same numeral twice in a count (Baroody & Price, 1983; Briars & Siegler, 1984; Frye, Braisby, Lowe, Maroudas, & Nicholls, 1989; Fuson, 1988; Miller, Smith, Zhu, & Zhang, 1995; Schaeffer et al., 1974; Wagner & Walters, 1982). Children also violate the stable-order principle, by producing different numeral lists at different times (Baroody & Price, 1983; Frye et al., 1989; Fuson, Secada, & Hall, 1983; Fuson et al., 1982; Miller et al., 1995; Wagner & Walters, 1982). These findings have led many observers to conclude that the how-to-count principles, rather than being understood from the outset, are in fact gradually learned. This is known as the *principles-after* (or *skills-before-principles*) view.

Of course, as Greeno, Riley, and Gelman (1984) point out, children might have trouble pointing to objects and reciting the list even if they *do* understand how counting represents number. Much more troubling for the principles-first view is evidence that young children do not understand the cardinal principle. That is, children do not seem to recognize that the last numeral used in counting tells the number of items in the set. One type of evidence comes from How-Many tasks. The version used by Schaeffer et al. (1974) is typical:

> "Each child was asked to count the chips in a line of *x* poker chips, where *x* varied between 1 and 7. After the child had counted the chips, the line was immediately covered with a piece of cardboard and the child was asked how many chips were hidden. Evidence that he knew... [the cardinal principle] was that he could respond by naming the last [numeral] he had just counted." (p. 360)

Some investigators have argued that the How-Many task overestimates children's knowledge, because some children actually do repeat the last numeral used in counting without (apparently) understanding that it refers to the cardinal value of the set (Frye et al., 1989; Fuson, 1988).

Conversely, it has been claimed that the How-Many task *under*estimates children's knowledge (Gelman, 1993; Greeno et al., 1984), because many children respond incorrectly to the question "how many," even after they have counted the array correctly. Rather than answering with the last numeral of their count, children who are asked "how many" usually try to count the set again. If they are prevented from recounting, they either make no response or give some numeral other than the last numeral of their count (Frye et al., 1989; Fuson, 1992; Markman, 1979; Rittle-Johnson & Siegler, 1998; Schaeffer et al., 1974; Wynn, 1990, 1992).

Supporters of the principles-first position argue that such behavior generally demonstrates that children *do* understand the cardinal principle. They point out that it is pragmatically strange to ask "how many" immediately after counting (Gelman, 1993; Greeno et al., 1984). To demonstrate this point, Gelman (1993) did a How-Many task with college students: "When we asked undergraduates a how-many question about 18 blocks, all of them counted but only one bothered to repeat the last count word said. Repeats of the question elicited puzzlement, some recounting, and so forth... " (p. 80).

In short, people disagree about whether the How-Many task underestimates, overestimates, or accurately measures children's knowledge of the cardinal principle. This raises other questions – namely, if the How-Many task doesn't test understanding of the cardinal principle, what does it test? And conversely, if cardinal-principle knowledge cannot be tested by the How-Many task, then how can this knowledge be tested?

[1] Words such as "one, two, three, . . ." etc. are commonly called "number words" in the psychological literature and "numerals" in the linguistics literature. We prefer "numeral" because it means a symbol for an integer, whereas "number word" could simply mean a word that conveys information about number (e.g., *few, many*, etc.)

1.1. The Give-N task

The Give-*N* task[2] provides a different way of measuring cardinal-principle knowledge. In this task, the child is asked to create a set with a particular number of items. For example, the experimenter might ask the child to "Give two lemons" to a puppet. Studies using this task have found that children are often unable to create sets for numerals that are well within their counting range. For example, many children who can count to "five" are not able to create sets of five objects. Thus, if cardinal-principle knowledge is tested using the Give-*N* task (rather than the How-Many task) children appear to acquire the cardinal principle relatively late, and only after mastering the other two counting principles.

Give-*N* studies have also yielded a new picture of how numerals are learned. It turns out that a child's performance on the Give-*N* task goes through a series of predictable levels, first reported in a longitudinal study by Wynn (1992) and supported by many cross-sectional studies since (Condry & Spelke, 2008; Le Corre & Carey, 2007; Le Corre et al., 2006; Sarnecka & Gelman, 2004; Sarnecka et al., 2007; Schaeffer et al., 1974; Wynn, 1990). These performance levels are found not only in child speakers of English, but also for Japanese (Sarnecka et al., 2007) Mandarin Chinese (Le Corre, Li, & Jia, 2003; Li, Le Corre, Shui, Jia, & Carey, 2003) and Russian speakers (Sarnecka et al., 2007).

The developmental pattern is as follows. At the earliest level, the child makes no distinctions among the meanings of different numerals. On the Give-*N* task, she may always give one object to the puppet or she may always give a handful, but the number she gives is unrelated to the numeral requested. A child at this level can be called a "pre-numeral-knower," for she has not yet assigned an exact meaning to any of the numerals in her memorized numeral list.

At the next level (which most English-speaking children reach by age 2-1/2 to 3 years) the child knows only that "one" means one. On the Give-*N* task, she gives one object when asked for "one," and she gives two or more objects when asked for any other numeral. This is the "one"-knower level.

Some months later, the child becomes a "two"-knower, for she learns that "two" means two. At that point, she gives one object when asked for "one," and two objects when asked for "two," but she does not distinguish among the numerals "three," "four," "five," etc. For any of those numerals, she simply grabs some objects and hands them over. This level is followed by a "three"-knower level, and some studies also report a "four"-knower level. Collectively, children at these levels have been termed "subset-knowers" (Le Corre & Carey, 2007; Le Corre et al., 2006) because although they have often memorized the numeral list up to "ten" or higher, they know the exact meanings for only a subset of those numerals.

After the child has spent some time (often more than a year) as a subset-knower, her performance undergoes a dramatic change. Suddenly, she is able to generate the right cardinality for numerals "five" and above. But whereas she progressed through the subset-knower levels gradually (learning "one," then "two," then "three,"...) she seems to acquire the meanings of the higher numerals ("five" through however high she can count) all at once. We call children at this level *cardinal-principle-knowers* (sometimes abbreviated *CP-knowers*).

Within-child consistency on a wide variety of tasks suggests that cardinal-principle-knowers differ qualitatively from subset-knowers. Most conspicuously, subset-knowers do not use counting to solve the Give-*N* task (even if they are explicitly told to count), whereas cardinal-principle-knowers do use counting – an observation that led Wynn, 1990; Wynn, 1992 to call subset-knowers "grabbers," and cardinal-principle-knowers "counters". But the differences do not end there. For example, a "two"-knower is, by definition, unable to give three objects when asked for "three." But a "two"-knower is *also*

(a) unable to fix a set when told, for example, "Can you count and make sure you gave the puppet three toys?... But the puppet wanted *three* – Can you fix it so there are three?" (Le Corre et al., 2006);
(b) unsure whether a puppet who has counted out seven items has produced a set of "seven" (Le Corre et al., 2006);
(c) unable to point to the card with "three" apples, given a choice between a card with three and a card with four (Wynn, 1992); and
(d) unable to produce the numeral "three" to label a picture of three items (Le Corre et al., 2006).

Cardinal-principle-knowers succeed across the board on these tasks. Such qualitative differences in the counting behavior of subset-knowers and cardinal-principle-knowers suggest that what ultimately separates the groups is not just the size of the sets they can generate. Rather, it is that cardinal-principle-knowers understand how counting works, whereas subset-knowers do not.

Because these two groups are separated by their knowledge of the cardinal principle, they offer us a way of finding out whether the How-Many task underestimates, overestimates, or accurately taps cardinal-principle knowledge. More importantly, they give us a way to explore the nature of cardinal principle knowledge itself.

1.2. Unpacking the cardinal principle

The cardinal principle is often informally described as stating that the last numeral used in counting tells how many things are in the whole set. If we interpret this literally, then the cardinal principle is a procedural rule about counting and answering the question 'how many.' If so, then cardinal-principle-knowers should answer the 'how many' question correctly (i.e., they should repeat the last word of a count) whereas subset-knowers should not. In

[2] This task is also known as "Give-a-number" (Condry & Spelke, 2008; Le Corre & Carey, 2007; Le Corre, Van de Walle, Brannon, & Carey, 2006; Sarnecka & Gelman, 2004; Sarnecka, Kamenskaya, Yamana, Ogura, & Yudovina, 2007; Wynn, 1990; Wynn, 1992); the "Verbal production task" (Baroody, 1987); "Make a set of N" (Fuson, 1988); and Give-me-X" (Frye et al., 1989). In a similar task, Schaeffer et al. (1974) asked children to tap a drum *n* times.

other words, the How-Many task should accurately tap cardinal-principle knowledge. The first question of the present study then, is: *Is the cardinal principle a procedural rule about counting and saying 'how many'?*

Alternatively, the cardinal principle can be viewed as something more profound – a principle stating that *a numeral's cardinal meaning is determined by its ordinal position in the list.* This means, for example, that the fifth numeral in any count list – spoken or written, in any language – must mean five. And the third numeral must mean three, and the ninety-eighth numeral must mean 98, and so on.

If so, then knowing the cardinal principle means having some implicit knowledge of the successor function – some understanding that the cardinality for each numeral is generated by adding one to the cardinality for the previous numeral. The second question of the present study then, is: *Is the cardinal principle a conceptual rule that is related to knowledge of the successor function?*

If children know how the numeral list instantiates the successor function, they should understand two things: (1) The *direction* of numerical change: In the numeral list, the word that denotes cardinality $N+1$ will come *after* the word denoting cardinality N. (2) The *unit* of numerical change: The word for cardinality $N+1$ must be the *very next word* in the numeral list, after the word for cardinality N.

1.3. The present study

In order to answer the first question (i.e., *Is the cardinal principle a procedural rule about counting and saying 'how many'?*) We devised a How-Many task that avoids the pragmatic oddness of asking how many items are in a set the child just counted. In our task, the experimenter counted a set the child could not see, and then asked the child how many items there were. This task allowed us to assess when children learn a procedural 'how-many' rule (i.e., a rule saying that the answer to the question 'how many' is the last word of a count) and whether mastery of this rule corresponds to understanding of the cardinal principle (as measured by the Give-*N* task).

To answer the second question (i.e., *Is the cardinal principle a conceptual rule that is related to knowledge of the successor function?*), we devised two tasks (the Direction task and the Unit task) to tap children's understanding of how the direction and unit of numerical change are represented by moving forward or backward along the numeral list. If knowledge of the cardinal principle is closely related to knowledge of the successor function, then cardinal-principle-knowers should succeed at these two tasks and subset-knowers should fail. These tasks are especially suitable because they involve addition and subtraction from sets, which according to Gelman and colleagues is the best way to reveal children's conceptual competence with respect to number representation (Cordes & Gelman, 2005; Zur & Gelman, 2004).

The strategy of the present study thus involved testing each child on six different tasks. First, the Give-*N* task was used to sort children into knower-level groups. Next, two tests of counting fluency (the Sequence and Correspondence tasks) were included to provide a baseline measure of the child's mastery of the numeral sequence and standard counting procedure. Then, our How-Many task was used to probe the child's understanding that the last word in a count sequence is the correct answer to a subsequent 'how many' question. Finally, two new tasks (the Direction and Unit tasks) tested whether children knew that (a) adding an element to a set requires going forward in the numeral list to represent the cardinality of the resulting set, whereas subtracting requires going backward (the Direction task) and (b) if one item is added, the resulting cardinality is named by *next* numeral in the list, whereas if two items are added, the resulting cardinality is named by the numeral after that (the Unit task).

2. Method

2.1. Participants

Participants included 73 children (28 boys, 45 girls), ranging in age from 2 years, 10 months to 4 years, 3 months (mean age 3–6). All children were monolingual and native speakers of English. Approximately half the children (38 out of 73) were recruited by mail and phone using public birth records in the greater Boston area. These children were tested at a university child development laboratory. Parents who brought their children in for testing received reimbursement for their travel expenses and a token gift for their child. The other 35 children were recruited and tested at university-affiliated or private childcare centers in Irvine, California. Parents received a token gift for their child when they signed up for participation; preschools received gift certificates to a children's bookstore and cognitive development seminars for their staff.

Families were not asked about their ethnicity, household income, or education, but participants were presumably representative of the middle-class, predominantly European-American and Asian-American communities in which they lived. The two samples (Massachusetts and California) did not differ significantly in age or in proportion of girls to boys.

Children were tested in one or two sessions, depending on their willingness to continue playing. In order for a child's data to be included, that child had to complete at least three of the six tasks in the study. Three additional children (two girls, ages 32 months and 40 months; one boy, age 38 months) began the study but quit before completing three tasks. These children's data were excluded.

2.2. Tasks

Give-N task (Frye et al., 1989; Wynn, 1990, 1992). The purpose of this task was to determine which numerals the child knew the exact meanings of. How a child performed on this task determined her 'knower-level' (i.e., "one"-knower, "two"-knower, cardinal-principle-knower, etc.). Materials for this task included a green dinosaur puppet (approx. 24 cm tall and 24 cm in circumference), a blue plastic plate (11 cm in diameter), and 15 small plastic lemons (approx. 2×3 cm each). To begin the task, the experimenter placed the puppet, plate, and lemons on the table and said, "In this game, you will give things to the dinosaur, like this." (The experimenter mimes placing something on the plate, then slides the plate over to the puppet.) Requests were of the form "Can you give *one* lemon to the dinosaur?" After the child responded to each request, the experimenter asked the follow-up question, of the form "Is that *one*?" If the child said "no," the original request was restated (e.g., "Can you give the dinosaur *one* lemon?"), followed again by the follow-up question (e.g., "Is that *one*?"). This continued until the child said that she had given the dinosaur the requested number of objects.

All children were first asked for one lemon, then three lemons. Further requests depended on the child's earlier responses. When a child responded correctly to a request for N, the next request was for $N + 1$. When she responded incorrectly to a request for N, the next request was for $N - 1$. The requests continued until the child had at least two successes at a given N (unless the child had no successes, in which case she was classified as a pre-numeral-knower) and at least two failures at $N + 1$ (unless the child had no failures, in which case she was classified as a cardinal-principle-knower).

The highest numerals requested were "five" and "six." It was important to include the numeral "five" because earlier studies have reported that children who can generate sets of five in the Give-*N* task are cardinal-principle knowers (Wynn, 1990, 1992; Le Corre & Carey, 2007; Le Corre et al., 2006). But early testing showed that children often generated sets of five just by grabbing a handful, because five was the number that would fit comfortably in one of their hands. Also, we reasoned that "four"-knowers might generate sets of five just by adding lemons to the plate until the number was bigger than they could name. For these reasons, we requested "five" and "six" in alternation (i.e., all children who received two cardinal-principle requests got one request for "five" and another for "six").

A child was credited with knowing a given numeral if she had at least twice as many successes as failures for that numeral. Failures included either giving the wrong number of items for a particular numeral N, or giving N items when some other numeral was requested. Each child's knower-level corresponds to the highest number she reliably generated. (For example, children who succeeded at "one" and "two," but failed at "three" were called "two"-knowers.) Children who had at least twice as many successes as failures for trials of "five" and "six" were called cardinal-principle-knowers.

2.2.1. Sequence task

This task measured the child's memorization of the numeral list up to "ten." To begin the task, the experimenter said, "Let's count. Can you count to ten?" If the child did not immediately start counting, the experimenter said, "Let's count together. One, two, three, four, five, six, seven, eight, nine, ten. OK, now you count." Each child's score reflects the highest numeral she reached without errors. For example, a child who counted "one, two, five" would have counted correctly to "two," and so would receive that score. Children were allowed to start over if they asked to, or if they did so spontaneously, but they were not told to start over by experimenters. For children who counted more than once, only their best count was used.

2.2.2. Correspondence task

This task measured the child's skill at tagging objects in one-to-one correspondence with the numeral list. Materials included two corkboards (36 cm × 12.5 cm) and a set of large, brightly colored push-pins (each 2.5 cm across). One corkboard had an array of five push pins, the other had ten; the pins were evenly spaced and arranged in a straight line. To begin the task, the experimenter presented the array of five and said "Now show me how you count these." This was followed by the array of ten. Each child's score reflects the highest number of items she was able to count without skipping or double-counting any item. No errors were allowed on the array of five; a maximum of one error was allowed on the array of ten (i.e., a child who made only one skip or double-count error on the array of ten still received a score of ten). Children were allowed to start over if they asked to, or if they did so spontaneously, but they were not told to start over. For children who counted an array more than once, only their best count was used.

2.2.3. How-Many task

This task probed children's responses to a 'how-many' question following a standard count by an adult. Materials for this task included three picture cards (approx. 23 cm × 7 cm) each depicting a row of items (three peppers, each 7 × 7.5 cm; five onions, each 5 × 5 cm; and seven tomatoes, each 2.5 × 2.5 cm). The task was introduced as follows: The experimenter sat across from the child and held up the picture of three peppers facing herself, so that the child could not see the picture. The experimenter said, "I have a picture of some peppers here, and I'm going to count them. You try to guess how many peppers are in the picture by listening to my counting. Ready? One, two, *three*. OK, how many peppers?" The first trial was a training trial. The experimenter showed the card to the child afterward and commented on the child's answer (e.g., "That's right! It *was* three peppers." or "Oops, it was actually *three* peppers. Good try, though."). The point of this training trial was to demonstrate how the game worked, and to show that the experimenter had counted in a standard way (i.e., that there was nothing 'tricky' about the counting).

Next, the test trials (using the pictures of five and seven items) were presented in counterbalanced order. On these trials, the child received mildly positive feedback (e.g., "OK!") regardless of her answer, and was not allowed to see the cards after answering.

2.2.4. Direction task

The purpose of this task was to find out whether the child understood that moving forward in the numeral list represents adding items to a set, whereas moving backward represents subtracting items. Materials included two clear plastic plates (approx. 23 cm in diameter) and four small plastic tubs, each containing 12 small objects, six of one color and six of another color (light blue and magenta hair bands, green and orange jacks, red and purple bears, yellow and dark blue dragonflies). Each set of items was used only once; order of item presentation was randomized by allowing the child to choose the items for the next trial.

The experimenter began each trial by placing either five or six items of the same color on each plate, saying for example, "OK, I'm putting FIVE bears on here [while placing five red bears on one plate]... and FIVE bears on here [while placing five purple bears on the other plate]... so this plate has five, and this plate has five." Then the experimenter moved one item from one plate to the other, saying, "And now I'll move one." (In this example, one of the plates would now contain four red bears, and the other would contain five purple bears and one red bear.) Next the experimenter would say "OK, now there's a plate with FOUR, and a plate with SIX. And I'm going to ask you a question about the plate with SIX. Are you ready? Which plate has SIX?" If the child attempted to count the items, the experimenter immediately covered up both plates and said "Oops! This isn't a counting game – this is a guessing game. So you can just guess." Each child received four trials, in randomized order: Two trials started with five items per plate, one trial asked about "four," the other about six"; the other two trials started with six items per plate, one trial asked about "five," the other about "seven." Each trial was scored correct or incorrect.

2.2.5. Unit task

The purpose of this task was to find out whether the child understood that the unit of numerical increase represented by moving from one numeral to the next on the list is exactly one item. Specifically, this task tests whether children know that moving forward one word in the list means adding one item to the set, whereas moving forward more than one word in the list means adding more than one item to the set. Materials for this task included a wooden box (17.5 × 12.5 × 5 cm) and six small plastic tubs. Each tub contained seven identical toys (frogs, bananas, worms, sea horses, fish, or rabbits). Each set of items was used only once; order of item presentation was randomized by allowing the child to choose the items for the next trial.

The experimenter began each trial by placing a number of items in the box, saying for example, "OK, I'm putting FOUR frogs in here." Then the experimenter closed the box and asked the memory-check question "How many frogs?" If the child did not answer the memory-check question correctly (e.g., "four"), the experimenter said, "Oops, let's try again" and repeated the beginning of the trial. After the child answered the memory-check question correctly, the experimenter said "Right! Now watch. .. "and added either one or two more items. Then the experimenter asked the test question, of the form, *now is it N + 1, or N + 2?* (e.g., "Now is it FIVE, or SIX?") Each child received two warm-up trials (1 + 1 item and 1 + 2 items) followed by four test trials (4 + 1, 4 + 2, 5 + 1, and 5 + 2) in counterbalanced order. For the trials beginning with one item, the test question was "Now is it TWO, or THREE?" For the trials beginning with four items, the question was "Now is it FIVE, or SIX?" For the trials beginning with five items, the question was "Now is it SIX, or SEVEN?" No feedback was given after any of the trials, although children could see the contents of the box when the experimenter opened it to return the items to their tub. Each trial was scored correct or incorrect.

2.3. Order of tasks

Order of tasks was randomized in the following way. The materials for each task were placed inside a large, opaque drawstring bag. (The Sequence, Correspondence, and How-Many tasks were grouped together in one bag.) Each bag was a different color (red, blue, green or yellow). At the beginning of the session, the child was asked, "What game should we play first?" and was given a choice of the four bags. The child chose a bag (without looking inside it), and the experimenter proceeded with the task in that bag. When the task was done, the child was allowed to choose from the three remaining bags, and so forth. For 42 of the children, all the tasks were presented in this way. The other 31 children completed the Give-*N* task during their first session, as part of a larger project, and completed the other five tasks (Sequence, Correspondence, How-Many, Direction, and Unit) during a second session no more than one week later (mean 4.6 days later.)

3. Results and discussion

3.1. Merging of Massachusetts and California samples

Initial analyses revealed no significant differences between the Massachusetts and California samples on any measure, so data were merged for the analyses reported here.

3.2. Give-N task

This task measured children's knowledge of the exact meanings of the numerals "one" through "six"; it was the basis for sorting children into knower-levels. Knower-level was significantly correlated with age, Kendall's *tau-b* = .397, $p < .0001$. Fig. 1 shows the age range for each knower-level.

Of the 73 children tested, 36 (49.3%) were cardinal-principle-knowers. These children ranged in age from 2–11 to 4–3 (mean age 3–8). The remaining 37 children (50.7%) were subset-knowers, so designated because they knew numerical meanings for only a subset of the numerals in their memorized count list. Subset-knowers ranged in age from 2–10 to 4–0 (mean 3–5). Breaking it down by knower-level, there were two pre-numeral-knowers (ages 2–11 and 3–3) and two "one"-knowers (ages 2–11 and 3–2); these groups were merged for analysis of performance on all other tasks ($n = 4$, mean age 3–1). There were 14 "two"-knowers (ages 2–10 to 4–0, mean 3–3); ten "three"-knowers (ages 2–11 to 4–0, mean 3–5); and nine "four"-knowers (ages 3–5 to 3–10, mean 3–7).

Although we did not explicitly tell children to count, we did record whether or not children spontaneously counted out loud, either when giving lemons to the dinosaur or when answering the follow-up question (*Is that N*?). As

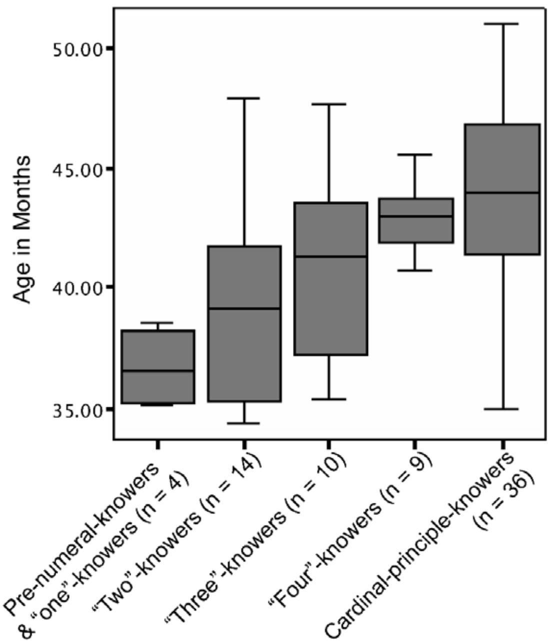

Fig. 1. Ages of subjects at each knower-level. (Knower-levels determined by Give-*N* task.) Boxes enclose the middle 50% of values; horizontal line in box indicates the mean for each group; extending lines indicate the top 25% and bottom 25% of values.

previous studies have reported, cardinal-principle-knowers counted much more often than subset-knowers, with 24 cardinal-principle-knowers (67%) counting aloud on at least one trial, whereas only six subset-knowers (16%) ever counted.

The distribution of counting across trial types is also informative. Subset-knowers were equally likely to count on low-number trials (i.e., trials asking for one, two, three, or four items) as on high-number trials (i.e., trials asking for five or six items). Specifically, two subset-knowers counted only on a single low-number trial; two others counted only on one or two high-number trials, and two counted on both types of trial. Interestingly, three of the four subset-knowers who ever counted on a high-number trial were "four"-knowers. (The other was a "three"-knower.) Thus, although these children tried to use counting to construct sets of five and six items, they were not able to use it successfully. (None of them succeeded on more than one high-number trial, and all of them failed at least two high-number trials, which is why they were classified as subset-knowers.)

The cardinal-principle-knowers, on the other hand, either used counting on the high-number trials only, or used it on both types of trial. (Unlike the subset-knowers, there was no cardinal-principle-knower who used counting only on low-number trials.) In fact, cardinal-principle-knowers were over six times more likely than subset-knowers to use counting on high-number trials – counting aloud on 67% of high-number trials, as compared to the subset knowers' 11%. (Setting "four"-knowers aside, only 1 out of 28 pre-numeral-knowers-through-"three"-knowers ever counted at all, making cardinal-principle-knowers over 22 times more likely to count than lower subset-knowers.)

Thus, our data find the same qualitative differences between cardinal-principle-knowers and subset-knowers as have been reported in earlier studies (Le Corre & Carey, 2007; Le Corre et al., 2006; Sarnecka & Gelman, 2004; Wynn, 1990, 1992).

3.3. *Sequence and Correspondence tasks*

These tasks measured participants' skill at producing the numeral list (Sequence task) and at pointing-and-counting arrays of objects (Correspondence task). On both tasks, children in all groups performed at or near ceiling, meaning that they recited the numeral list up to ten, and also pointed to each object in an array once and only once.

The mean score of all children on the Sequence task was 9.94 (range 8–10). The mean and range of scores for each group (in order, beginning with the pre-numeral-knowers/"one"-knowers) was 10.00 (range 10–10); 9.75 (range 8–10); 9.90 (range 9–10); 10.00 (range 10–10); and 10.00 (range 10–10), respectively.

The mean score of all children on the Correspondence task was 9.32 (range 1–10). The mean and range for each group (in order, beginning with the pre-numeral-knowers/"one"-knowers) was 8.75 (range 5–10); 9.07 (range 4–10); 8.80 (range 3–10); 9.44 (range 5–10); and 9.61 (range 1–10), respectively. The median and modal scores for each group on both tasks were 10.0.

All the children except two produced the numeral list up to "ten" on at least one counting task. The exceptions were one "two"-knower (age 2–10) who produced the sequence up to "eight," and one "three"-knower (age 4–0) who produced the sequence up to "nine." Thus, every child was familiar with the portion of numeral sequence (i.e., "four-five-six-seven") relevant to the other tasks in this study.

3.4. *How-Many task*

The How-Many task measured how often children answered a the question"how many" with the last numeral of the experimenter's count. This was an open-ended task – a child could answer with the correct numeral, with a different numeral, or with no numeral at all. Since children knew at least ten numerals, if they were providing a numeral response but otherwise answering at random, chance would conservatively be 10%.

On the training trial (where the experimenter counted to three), 65 children (76%) correctly answered "three"; seven children (21%) produced some other numeral (answers ranged from "one" to "ten"), and one child (a "one"-knower) began counting out loud, starting at "one" and continuing until the experimenter stopped him at "twelve."

After feedback was given on the training trial, each child received two test trials, (where the experimenter counted to five and seven) in counterbalanced order. Overall, children answered correctly 83% of the time. On the other 17% of trials, children either responded with an incorrect numeral (13% of total trials) or counted aloud (4% of total

Table 1
Results of How-Many task

Knower-level	*n*	Both trials correct	1 correct, 1 incorrect	Both trials incorrect
Pre- & "One" – Knowers	4	0	2	2
"Two" – Knowers	10	6	0	4
"Three" – Knowers	10	8	2	0
"Four" – Knowers	7	5	1	1
CP-Knowers	36	34	1	1

trials). Children who counted always began at "one" and continued on past the target numeral – the exception was one trial where the experimenter had counted to "five" and the child also counted to "five" (i.e., repeated the experimenter's count verbatim). This child counted to "five" on the trial of seven as well.

Breaking down performance by knower level: Cardinal-principle-knowers almost always answered correctly (96% of trials). This was significantly higher than the subset-knowers' overall success rate of 68%, $t(67) = 3.53$, $p = .001$; see Table 1. However, all subset-knowers did not perform alike. On the contrary, the data in Table 1 show that the most dramatic difference was between the pre/"one"-knowers and the "two"-knowers (25% correct and 64% correct, respectively). For "two"-knowers and above, the success rate is always over 60%. Thus, it appears that the answer to our first question (*Is the cardinal principle a procedural rule about counting and saying 'how many'?*) is *no.* Whatever knowledge allows children to succeed on the How-Many task, it is different from the cardinal principle.

It is also apparent from Table 1 that most children either got both trials right (53 children) or got them both wrong (8 children). It was relatively rare for a child to get one trial correct and the other incorrect. Of course, the cardinal-principle-knowers usually got both trials correct, but even among the subset-knowers, only 5 children got 1 trial right and 1 trial wrong, whereas 26 children got both trials right or wrong. This error pattern makes sense if there is a rule (i.e., a procedural How-Many rule) that has been learned by some children and not others. It is not the pattern one would expect to see if all participants understand the relevant concepts but sometimes did not apply them because of procedural error. A constant, low level of error would make it much more likely for any given child to get *one* trial incorrect than to get *both* trials incorrect.

To formally test these intuitions, we asked the following questions:

(1) Does the rate of correct answers increase with knower-level or stay the same?
(2) Is the rate of correct answering for each child essentially dichotomous (i.e., their probability of giving a correct answer is either very high or very low, so that they either get both right or none right)?

We developed four models that expressed all four combinations of these two possibilities, and estimated the Bayes factors between these four models (to decide which model best fit the data) using the computational method known as 'reverse jump Markov chain Monte Carlo'. Results are given in Table 2. It is clear that the Dichotomous-Increasing model is by far the best one, with all of the Bayes factors exceeding the suggested scientific standards for 'very strong' evidence (see Kass & Raftery, 1995, p. 777). In other words, this analysis suggests that children either knew or didn't know a How-Many rule, and that their likelihood of knowing it increased with knower-level.

Table 2
Bayes factors between the four models, relative to the most likely Dichotomous-Increasing model

Model	Bayes factor	Log Bayes factor
Dichotomous-Increasing	1	0
Dichotomous-Same	9×10^8	20.63
Continuous-Increasing	1,966	7.58
Continuous-Same	3×10^7	17.21

Note. Bayes factors can be though of as betting ratios – e.g., the Continuous-Increasing model is 1966 times less likely to be generated by the current data than the most likely Dichotomous-Increasing model. The Log Bayes factors are included because they may be more familiar to some readers.

Thus, we see that many young children have formulated the generalization that the last word reached in a count is the appropriate answer to the question "how many." Furthermore, they apply this rule long before they demonstrate any understanding of the cardinality principle on tasks such as Give-*N*, that do not use the exact phrase *how many*.

3.5. Direction task

This task was designed to measure children's understanding that moving *forward* in the numeral list represents *adding* items, whereas moving *backward* represents *subtracting* items. Only two children ever tried to count the items; they were prevented from doing so as described in the method section, above.

Initial analyses found no significant difference among the lower knower-levels (pre-numeral-, "one,"- "two"- and "three"-knowers), so these levels were merged in the following analysis. A multivariate ANOVA controlling for age revealed a significant main effect of knower-level group (lower knower-levels/"four"-knowers/cardinal-principle knowers) on Direction scores, $F(3) = 3.13$, $p = .027$; observed power = .721. Independent samples *t*-tests[3] found that the lower knower-level group performed significantly worse than either "four"-knowers, $t(27) = 2.67$, $p = .01$; or cardinal-principle knowers, $t(53) = 2.08$, $p = .04$. "Four"-knowers and cardinal-principle knowers did not differ from each other, $t(28) = .439$, p = N.S. One-sample *t*-tests based on the original hypotheses found that pre/"one"-knowers, "two"-knowers and "three"-knowers all performed at chance, whereas "four"-knowers and cardinal-principle-knowers performed above chance, $t(7) = 3.42$, $p = .01$ and $t(29) = 2.29$, $p = .03$, respectively (see Fig. 2).

[3] Equal variances were not assumed for any measure.

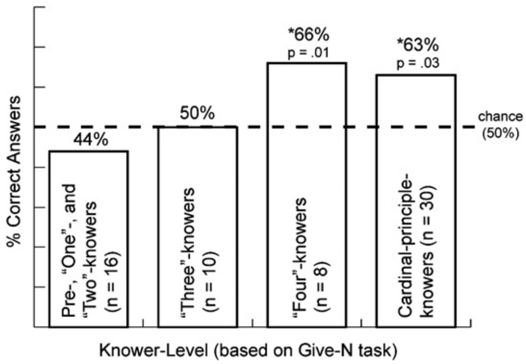

Fig. 2. Direction task. Only "four"–knowers and cardinal-principle-knowers performed above change on the Direction task, designed to test their understanding that going forward in the numeral list corresponds to adding items, and going backward corresponds to subtracting items.

3.6. Nonparametric tests

A Kruskal–Wallis test found a significant effect of knower-level group on Direction scores, *chi*-square (2) = 6.34, $p = .04$; Mann–Whitney comparisons found that "four"-knowers and cardinal-principle-knowers each performed significantly better than pre- through "three"-knowers, $Zs > 2.14$, $ps < .05$; but that the two higher groups did not differ from one another.

How did children solve the Direction task? We designed the Direction task to tap implicit knowledge of the successor function. But a child who can subitize (i.e., recognize without counting) sets of four items – and this should include all "four"-knowers and cardinal-principle-knowers – could solve the four-versus-six trials simply by recognizing the plate with four items. This 'recognize four' strategy would allow children to succeed on the four-versus-six trials, but wouldn't affect the five-versus-seven trials. (Five is outside the subitizing range – that is, it is too high a number to be recognized without counting.)

A comparison of the two trial types, however, does not support the hypothesis that "four"-knowers or cardinal-principle knowers succeeded on this task by using a 'recognize four' strategy. A repeated measures ANOVA controlling for age showed no interaction of knower-level group (lower knower-levels/"four"-knowers & CP-knowers merged) by trial type (four/six trials versus five/seven trials), $F(1) = .069$, p = N.S. Paired *t*-tests also revealed no significant effect of trial type, either for the "four"-knowers and cardinal-principle-knowers merged, or for either group alone.

3.7. Distribution of direction scores

In the Direction task, unlike the How-Many task, it was not the case that children either got all the trials right or performed at chance. What is clear is that the task was a difficult one; although "four"-knowers and cardinal-principle knowers as a group succeeded on the task, quite a few individual children in each group performed at chance (which was 50% in this task, see Fig. 3). We believe that this task was difficult for children because it taps knowledge that is new and fragile. But because we saw no way to do small-number warm-up trials for the Direction task (with the sets visible, children could have solved the small-number trials without reasoning about the successor function at all), we cannot rule out the possibility that children were confused by it for other reasons. For example, they might not be able to simultaneously represent a subtraction from one set and an addition to the other.

Overall though, we take these results to mean that part of what separates cardinal-principle-knowers from subset-knowers is an understanding of the mapping between (a) the direction of movement along the numeral list and (b) the direction of change in the numerosity of a set. Nevertheless, this can't be the whole story, because "four"-knowers succeed at the Direction task, but still do not use counting to solve the Give-*N* task (which is why they aren't cardinal-principle-knowers). There must be some other piece of knowledge – something that only cardinal-principle-knowers know.

3.8. Unit task

This task measured children's understanding that going forward *one* word in the number list means adding *one* item, whereas going forward *two* words means adding

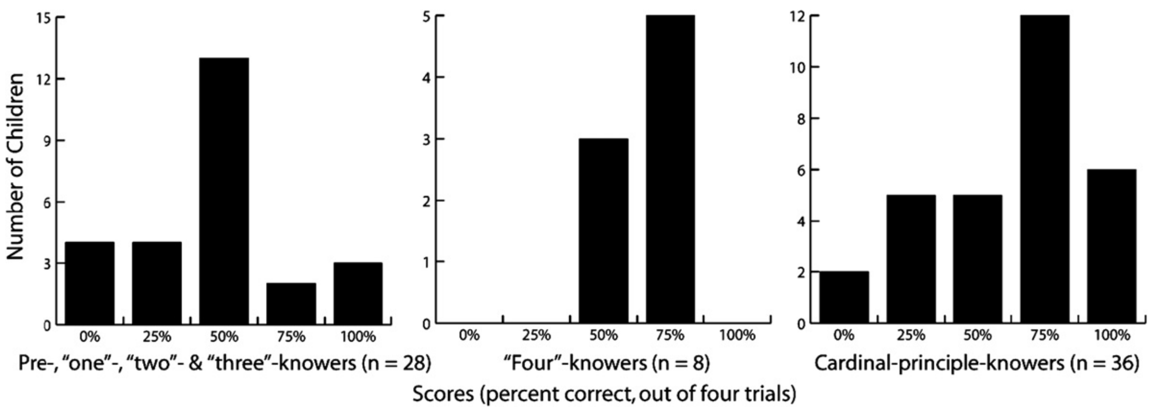

Fig. 3. Distribution of scores on the Direction task. Although many children in each group performed at change (50%), it was *not* the case that most children either performed at chance or performed perfectly.

two items. In the first two (warm-up) trials, children were asked to judge whether a box that started with one item, and gained one or two more, had two items or three. Pre-numeral- and "one"-knowers performed at chance (50%); every other group performed significantly above chance, *ps* < .05. (The mean scores for each group were: "two"-knowers, 79% correct; "three"-knowers, 81% correct; "four"-knowers, 100% correct; CP-knowers, 89% correct.) These results from the warm-up trials indicate first, that children were able to understand the directions and do the task, and second that children at and above "two"-knower level have concepts of "one" and "two" that support the inferences required by this task. (As indeed we would expect them to.)

On the four test trials (4 + 1, 4 + 2, 5 + 1, 5 + 2), initial analyses found no significant difference among the lower knower-levels (pre-numeral &"one"-knowers, "two" -knowers, "three"-knowers), so these levels were merged in the analysis. A multivariate ANOVA controlling for age found a significant main effect of knower-level group (lower knower-levels/"four"-knowers/cardinal-principle-knowers) on Unit scores, $F(3) = 3.51$, $p = .022$; observed power = .748. Independent samples *t*-tests found that the cardinal-principle-knowers performed significantly better than either the lower knower-level group, $t(47) = 3.01$, $p = .004$, or the "four"-knower group, $t(10) = 2.98$, $p = .01$. The lower knower-level group and "four"-knowers did not differ from each other, $t(12) = .891$, p = N.S. One-sample *t*-tests based on the original hypotheses showed that pre-numeral & "one"-knowers, "two"-knowers, "three"-knowers and "four"-knowers all performed at chance, whereas cardinal-principle-knowers performed significantly above chance, $t(28) = 4.61$, $p < .001$. (See Fig. 4.)

3.9. Nonparametric tests

A Kruskal-Wallis test also found a significant effect of knower-level group on Unit scores, *chi-square* (2) = 12.37, $p = .002$; Mann–Whitney comparisons showed that cardinal-principle-knowers both performed significantly better than either "four"-knowers or pre- through "three"-knowers, *Zs* > 2.69, *ps* < .009; and that the two lower groups did not differ from one another.

As with the Direction task (and unlike the How-Many task), the distribution of scores on the Unit task was relatively normal, rather than bimodal. (See Fig. 5.) In other words, it was not the case that children either got all the trials right or performed at chance. This task was also a difficult one (recall that CP-knowers did not perform perfectly even on the warm-up trials – their mean score was 89% correct). The test trials made the same demands as the warm-up trials, and required knowledge of the successor function as well. Considering these challenges, it is unsurprising that even the cardinal-principle-knowers scored well below ceiling. Their success was not perfect; but as a group they did succeed.

These results suggest that cardinal-principle-knowers are separated from subset-knowers by their understanding of the *unit* of mapping between numerals and numerosities. Only cardinal-principle-knowers understand that going forward *one* word means adding *one* item; going forward *more* than one word means adding *more* than one item.

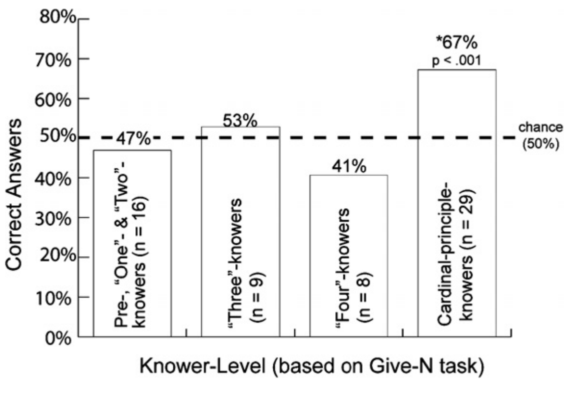

Fig. 4. Unit task. Only cardinal-principle-knowers had any understanding that going forward *one* word in the number list corresponds to adding *one* item, whereas going forward more than one word corresponds to adding more than one item.

4. General discussion

The jumping-off point for the present study was the (now well-established) observation that if you ask a two- to four-year-old child to give you various numbers of items (e.g., give me *two* blocks/give me *one* book/give me *four* crayons, etc.), the child's responses will fit the pattern for one of six knower-levels: Pre-numeral-knowers, "one"-, "two"-, "three"-, or "four"-knowers (i.e., subset-knowers), or cardinal-principle-knowers. Performance at different knower-levels varies along two parameters: (1) the set sizes children can generate upon request, and (2) whether or not they use counting to do so.

Why is it that some children (subset-knowers) can generate sets for only a few of the numerals in their count list (or none of them, in the case of pre-numeral-knowers), while others generate sets for all the numerals we tested? And why is it that only the latter group uses counting (which seems the obvious way to solve the problem)? The present study brings new data to bear on these questions. We replicate relevant findings from the literature and also present new findings from novel tasks designed specifically to determine what it is, exactly, that cardinal-principle-knowers know.

First, we sorted children into knower-levels, based on their ability to create sets of one to six items upon request. Our data confirmed previously published descriptions of knower-levels, both in the set sizes children could generate and in the fact that only cardinal-principle-knowers regularly used counting to solve the problem.

Second, we tested children on two baseline measures of counting skill: Counting out loud up to ten (the Sequence task) and counting arrays of five and 10 objects (the Correspondence task). Of the 73 children tested, 71 counted to ten and every child counted to at least eight, indicating that they had mastered the numeral sequence from four through seven (which was the part of the sequence needed for the other tasks in the study).

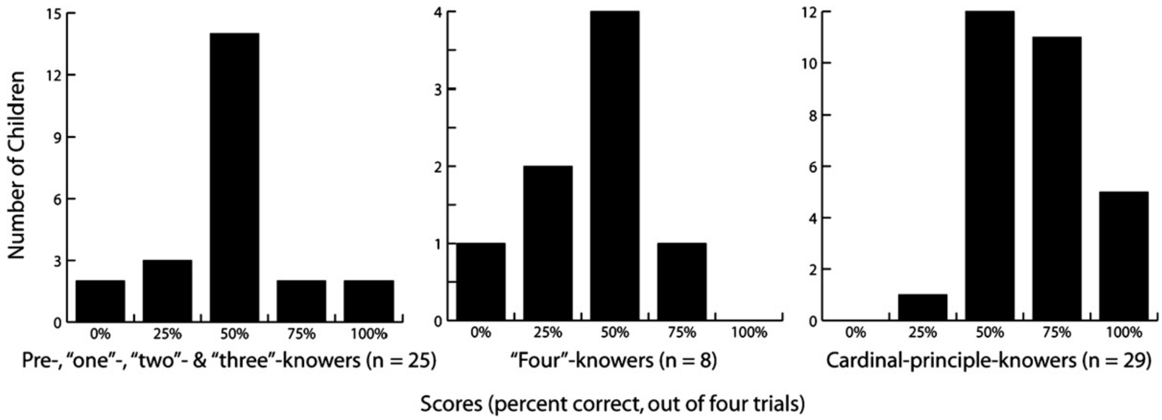

Fig. 5. Distribution of scores on the Unit task. Although many children in each group performed at chance (50%, or two out of four trials correct), it was *not* the case that most children either performed at chance or performed perfectly.

Third, we devised a task to probe for a procedural How-Many rule. In our task, the experimenter counted a set the child could not see and then asked the child to guess how many things were in it. If a child chose a numeral from her count list at random, her chance of responding correctly was 10% or less, depending on how many numerals she knew. (We only tested one through ten.) Thus, children were unlikely to get both of our test trials correct by chance. But virtually all of our cardinal-principle-knowers did get both trials correct, and so did three-quarters of the "two," "three"- and "four"-knowers, and a quarter of the pre-numeral-knowers. Clearly, many children learn a procedural 'how-many' rule (i.e., that the answer to the question "how many" is the last word reached in a count) long before they understand the cardinal principle. These findings confirm earlier reports (e.g., Frye et al., 1989; Fuson, 1988) that How-Many tasks should not be used as a measure of cardinal-principle understanding, because they tend to overestimate children's knowledge of how counting works.

One question that can be asked about these findings is whether it is unfair to compare children's performance on the How-Many task with their performance on Give-*N* (A.J. Baroody, personal communication, August 13, 2007). Previous studies (Baroody, 1987, 1999; Fuson, 1988, 1992) have differentiated between a *count-cardinal* rule, which says that the last word of a count sequence names the associated cardinality (this knowledge would be tapped by the How-Many task) and a *cardinal-count* rule, which says that the cardinal numeral for a set predicts the last word of a count sequence for that set (this knowledge would be tapped by the Give-*N* task.) Several studies have reported that the *count-cardinal* rule is learned earlier than the *cardinal-count* rule, which could provide an alternative explanation for the present findings.

We are inclined to doubt this explanation, because of the results reported by Le Corre et al. (2006) in their Counting Puppet task, which cardinal-principle knowers pass and subset-knowers fail. In Le Corre's study, children were told that a character wanted, for example, six cookies. A puppet then counted out five cookies and the child was asked "is that six?" Like our How-Many task, the Counting Puppet task requires the child to listen to a standard (not tricky or unusual) count, and to make a judgment about the result of that count. The main difference is that our How-Many task uses the specific phrase "how many" in the test question, whereas the Counting Puppet task does not. There is no obvious reason why either task should be a better test of the *count-cardinal* rule than the other, and no obvious reason why subset-knowers should succeed at our task and fail at Le Corre's, if the *count-cardinal* rule were the issue.

Given that other studies have shown high within-child consistency across cardinality tasks including Give-*N*, the Counting Puppet task, What's-On-This-Card, etc. (Le Corre et al., 2006; Le Corre & Carey, 2007; Wynn, 1992), it seems fair to conclude that the difference we find between How-Many scores and Give-*N* scores in the present study is a result of our task using the exact phrase "how many," which is enough to prompt many children to answer with the last word of a count, whether they understand the cardinal principle or not.

Although a 'how-many' rule is not the cardinal principle, it is interesting nevertheless. Three informative findings from the present study's How-Many task were that (a) the great majority of children either got both trials correct or neither trial correct, indicating that they either knew the rule or didn't know it; (b) most children had learned the rule by the time they were "two"-knowers; (c) when children answered incorrectly, they either produced a different numeral or (less commonly) produced the numeral list itself (i.e., counted out loud).

These facts suggest that children's interpretation of the phrase "how many" changes as their understanding of counting grows. The earliest interpretation is probably that "how many" is a prompt to count (i.e., to produce the numeral list that they have memorized – see Fuson, 1988 for related findings). Our How-Many results suggest that children soon figure out that "how many" is a question to be answered with a numeral, but children initially do not know how to decide which numeral it should be. Most "two"-knowers and above know that "how many" should be answered with the last numeral of a count. Despite all this, it will be some time before they understand how or why the last numeral of a count denotes the numerosity of the set.

Finally, the present study explored children's knowledge of how counting implements the successor function, and related this understanding to other measures of mastery of the cardinal principle. The Direction and Unit tasks probed whether children knew that if you start with a set of "six" and add one, the resulting set has "seven," not "five" (the Direction task) and the resulting set has "seven," not "eight" (the Unit task). Only cardinal-principle-knowers succeeded at both tasks, although their performance was far from perfect. Pre-numeral-, "one"-, "two" and "three"-knowers utterly failed both tasks. "Four"-knowers succeeded at the Direction task but failed the Unit task. From these results, we concluded that cardinal-principle-knowers have some implicit (albeit fragile) knowledge of how counting implements the successor function, whereas subset-knowers as a group do not.

The failure of the subset-knowers on the Unit and (except for "four"-knowers) Direction tasks is particularly important because these were both arithmetic tasks. Individual items were added or subtracted from sets and the child was given a forced-choice decision as to the cardinal value of the resulting set. Supporters of the principles-first position (Cordes & Gelman, 2005; Zur & Gelman, 2004) have argued that arithmetic tasks elicit the highest level of numerical understanding in preschool children, and that because Give-*N* is not an arithmetic task, it underestimates children's understanding of how counting represents number. However, the present study finds within-child consistency between Give-*N* and the Direction and Unit tasks, just as other studies have found within-child consistency between Give-*N* and other tests of cardinal-principle understanding.

The success of "four"-knowers on the Direction task might be thought of in two ways. First, some children classified as "four"-knowers on this task may well have had some fragile understanding of the cardinal principle. "Four"-knowers are relatively rare in the literature (e.g., of the 87 2- to 4-year-old children in Le Corre et al.'s (2006) studies, 8% were pre-numeral-knowers, 15%, 18%, and 20% were "one"- "two"- and "three"-knowers respectively, and 32% were cardinal-principle/CP-knowers, but only 7% were "four"-knowers), suggesting that children are "four"-knowers for only a short time before completing their construction of the cardinal principle.

Second, working out how counting represents number is not accomplished in a single step. Children learn the exact cardinal meanings of "one," "two," "three" and "four" before they become cardinal-principle knowers, and the present studies show that they learn a *how-many* rule as well. Sarnecka and Gelman (2004) showed that subset-knowers understand that numerals depict *some* precise cardinal value, before they know how to determine which value. It makes sense that children would work out the direction of numerical change before the unit of change, for the latter presupposes the former. The present studies suggest that finally putting together the puzzle of how counting implements the successor function is indeed what turns a subset-knower into a cardinal-principle-knower, but that "four"-knowers have almost all the pieces in place.

The present study has tried to characterize some of the partial knowledge children have as they figure out how counting implements the successor function. We have also identified subcomponents of cardinal-principle knowledge itself (the direction of change versus the unit of change) and shown that children learn these subcomponents, one before the other. Characterizing children's knowledge at various points in this process will in turn constrain the theories we build, as we attempt to understand an intellectual achievement that is as remarkable as it is commonplace – the young child's discovery of how verbal numerals represent the natural numbers.

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